in 1985 when the first experimental reports of coherent backscattering came in. This phenomenon is now successfully explained in terms of constructive interference between two waves propagating in opposite directions. New phenomena have also been found for the coherent beam and the speckles.

1.4.1 Diffuse Beam: Coherent Backscattering and Localization

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On the basis of reciprocity, interference between two opposite paths can be argued to be constructive in the backscattering direction of, for instance, a slab geometry, and *exactly* as large as the conventional diffuse background calculated from (1.8). At backscattering, the equation of radiative transfer is thus 100 % wrong! As always, the width of an interference effect is roughly given by the wavelength divided by the typical distance between two typical points of scattering, in this case the mean free path, giving $\Delta\theta \approx 1/k\ell$ [113]. One can still argue as to what mean free path should be used here: the transport or the scattering mean free path. Although a physical argument favors the first (recall Fig. 1.2), a rigorous confirmation for anisotropic scatterers (for which both mean free paths differ) has only been given recently [114, 115]. Thus

$$\Delta\theta \approx \frac{1}{k\ell^*} . \tag{1.17}$$

The smallness of $1/k\ell^*$ in typical experiments probably explains why the serendipitous discovery of coherent backscattering was unlikely (Fig. 1.4).

Coherent backscattering has been investigated in a variety of circumstances. The general reciprocity relation that can be written down between the transition matrix (relating the incoming and outgoing electric fields of the light) of any event, D, and that for the same event in the opposite sequence, R, placed in a magnetic field $\mathbf{B}_0$, is [22]

$$D(\sigma, \mathbf{k} \to \sigma', \mathbf{k}' \,|\, \mathbf{B}_0) = R(\sigma', \mathbf{k}' \to \sigma, \mathbf{k} \,|\, -\mathbf{B}_0) , \tag{1.18}$$

where σ ($= \pm$) indicates the two possible states of circular polarization. In the absence of a magnetic field one can verify that $D(\sigma, \mathbf{k} \to \sigma, -\mathbf{k}) = R(\sigma', \mathbf{k} \to \sigma, -\mathbf{k})$. This means that for the diagonal channel $\sigma = \sigma'$ the inverse scattering sequence has the same scattering amplitude as, and therefore interferes constructively with, its opposite partner. More precisely,

$$|R + D|^2 = |R|^2 + |D|^2 + 2 \text{ Re } RD^* = 2(|R|^2 + |D|^2)$$

at backscattering. This argument leads to the famous and apparently universal factor of two for the diagonal polarization channel. Absorption is allowed and therefore does not change this conclusion. Reciprocity does not inform us about the off-diagonal helicity channel. Experiments [116] and calculations [117–119] give a value of only 1.12 for this channel.

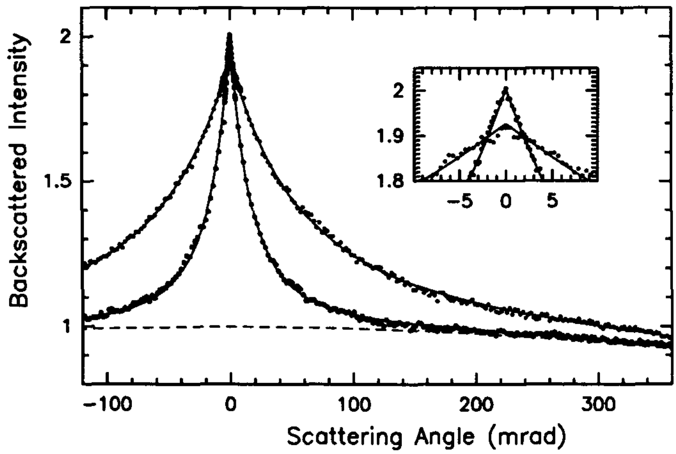

Fig. 1.4. A measurement of optical coherent backscattering from a dielectric random medium, performed with circularly polarized light with a vacuum wavelength of 633 nm. The backscattered intensity is plotted against the scattering angle, where zero corresponds to exact backscattering. The sample is $BaSO_4$ with a transport mean free path of roughly 2.1 μm. The high quality of this measurement was made possible by a new technique [122]. The measurement clearly shows the nonanalytic cusp in the center caused by extreme long-range diffusion [reprinted from Wiersma, Van Albada, van Tiggelen and Lagendijk, Phys. Rev. Lett. **74**, 4193 (1995), with permission from the American Physical Society]

The relation $D = R$ at backscattering can no longer be obtained from (1.18) once a magnetic field is present. The Cauchy inequality indicates that the factor of two can only shrink. In Fig. 1.5 we demonstrate coherent backscattering curves obtained by Erbacher, Lenke and Maret in an external magnetic field [21]. In a magnetic field the electric polarization vector is subject to Faraday rotation. This experiment proves that Faraday rotation kills the constructive interference, as allowed by reciprocity. These observations are in rather good agreement with calculations and numerical simulations carried out by Martinez and Maynard [22] (Fig. 1.6). The decrease of the enhancement factor seems to be a universal function of $VB\ell^*$, where V is the Verdet constant of the medium describing the rotation angle per tesla, per meter. So far, the equi-intensity lines of the scattering cone have been found to be circular, as expected if the direction of the magnetic field is along the backscattering direction. In order to verify a recent prediction [23] that these lines become elliptical because the diffusion tensor becomes a nondiagonal tensor in a magnetic field, one has to change the direction of the magnetic field.

Coherent backscattering has also been studied in relation to gain (using scatterers containing dye) [19], as well as in relation to internal reflection on the boundaries of the medium [120, 121]. Neither one of these mechanisms is believed to break the basic reciprocity argument leading to the factor of

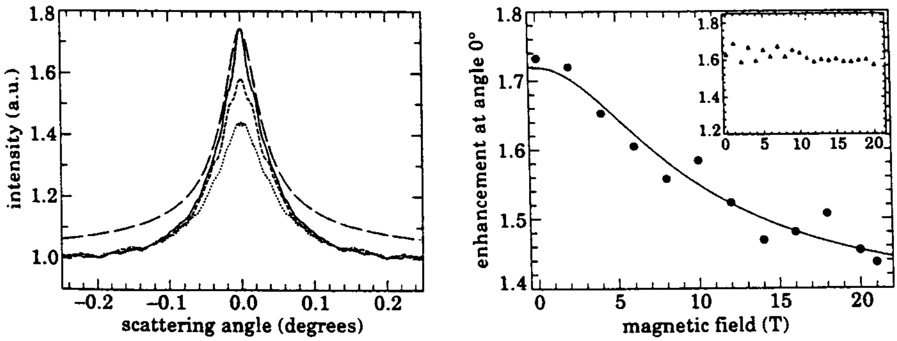

Fig. 1.5. Coherent backscattering in an external magnetic field. *Left:* angular dependence of the scattered light intensity in the vicinity of backscattering and for circular polarization, as obtained by azimuthally averaging video pictures around the peak position, and normalized to the flat background at 0.33°, in different magnetic fields: 0 T (*solid line*), 10 T (*short-dashed line*) and 21 T (*dotted line*). The *long-dashed line* corresponds to a fit to theory. The sample – 40 vol-% FR5 (a rare-earth-doped paramagnetic Faraday rotator glass), milled to a powder – had a length $L = 2$ mm and a transport mean free path $\ell^* \approx 70\,\mu$m. *Right:* magnetic-field dependence of the backscattered light intensity at exact backscattering of the same sample. The continuous line is a fit to theory. The inset shows a reference sample with negligible Faraday rotation but with same cone width [reprinted from Erbacher, Lenke and Maret, Europhys. Lett. **21**, 55 (1993), with permission from Les Editions de Physique, France]

two. The angular shapes may change however, because in both cases events involving long scattering paths will be favored, leading to a narrowing of the peak. In Fig. 1.7 we show measurements of the cone carried out in Amsterdam with gain [19]. These measurements confirm the picture above. The study of gain in combination with multiple scattering comes into the picture because it may offer the possibility of a random laser once the gain exceeds a critical threshold [123–125].

Does reciprocity really lead to an enhancement of exactly two in the helicity-conserving channel at backscattering? Indeed, if one applies the equation of radiative transfer and adds the coherent backscattering phenomenon required by reciprocity, this factor of two follows. However, this procedure violates flux conservation since the flux of the cone, though small and of order $1/k\ell$, is added ad hoc and is energetically not accounted for. At present a practical transport equation obeying both reciprocity and flux conservation is not available. A recent experiment done by Wiersma et al. in Amsterdam demonstrated for the first time the need for such an equation [36]. This experiment used samples with a value of $k\ell^* \approx 5$, and showed an enhancement factor lower than the "holy" factor of two predicted by the conventional approach for the helicity-conserving channel of circularly polarized light. The measured enhancement factor turned out to depend on the density of the scatterers, which excludes the alternative explanation of nonsphericity of the

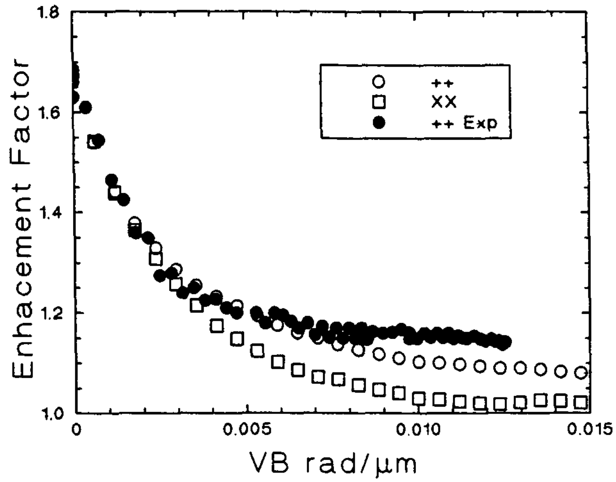

Fig. 1.6. Numerical Monte Carlo simulation of coherent backscattering in the presence of Faraday rotation. The experimental points were obtained by Erbacher, Lenke and Maret [21]. The numerical simulations were carried out by Martinez and Maynard [22] and show the coherent-backscattering enhancement factor in the ++ circular-polarization channel and in the xx linear-polarization channel. The input parameters for the numerical simulation were: $\lambda_0 = 0.4579$ μm, Mie particle radius $a = 0.1$ μm, and indices of refraction of 1.45 for the particle and 1.65 for the surrounding medium. The Verdet constant in the medium was $V = 1571$ rad/(mT) and was assumed to be zero inside the scatterers. In the experiment the Verdet constant is known to depend on the magnetic field. Experimentally, $L/\ell^* = 500$ [reprinted from Martinez and Maynard, Phys. Rev. B **50**, 3714 (1994), with permission from the American Physical Society]

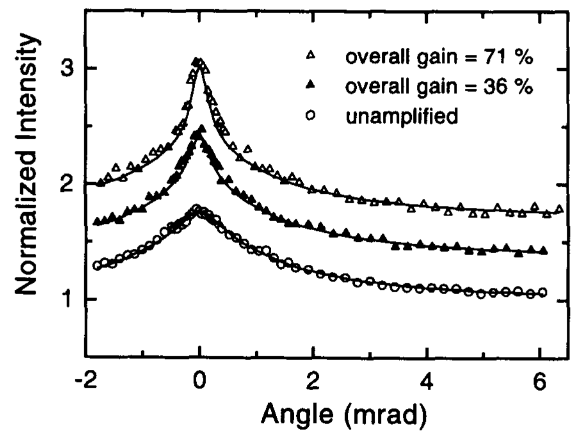

Fig. 1.7. Coherent backscattering in a medium with gain. As the gain of the medium increases, long paths of the photons achieve more weight and the coherent backscattering peak narrows [reprinted from Wiersma, Van Albada and Lagendijk, Phys. Rev. Lett. **75**, 1739 (1995), with permission from the American Physical Society]

particles. Only particles with rotational symmetry have a vanishing single-scattering signal in the helicity-conserving reflection channel. Since, in single scattering, only particles in the skin layer one mean free path in thickness contribute, the signal would be proportional to the particle density times the mean free path, i.e. independent of density. The experimental results could be reasonably explained by recurrent scattering of light between two particles, which can be seen as a sort of "super single scattering" from particles without rotational symmetry, leading to a density-dependent cross-section. This explanation also restores energy conservation to second order in the particle density [126].

The combination of flux conservation and reciprocity leads to a self-consistency problem that touches the heart of microscopic theories of strong localization [127]. The point is that interference not only modifies reflection in backscattering, but also changes the path-length distribution *inside* the system, and thus the background intensity. As a matter of fact the whole concept of "path-length distribution", a widely used term to refer to the contribution of "photon paths" of a particular length to the measured intensity, breaks down when interference is allowed. This complicated problem has so far only been considered in the diffusion approximation. In this theory one calculates the interference contributions to the diffusion constant (1.4), thereby requiring reciprocity and flux conservation. The result is, in three dimensions [127],

$$\frac{1}{D} = \frac{1}{D_{\mathrm{B}}} + \frac{1}{D}\frac{1}{4\pi k^2 \ell}\int_{q_{\min}}^{q_{\max}} \mathrm{d}^3\mathbf{q}\,\frac{1}{q^2}\,. \tag{1.19}$$

D_{B} is the diffusion constant without interference, $q_{\min}$ is a lower cutoff related to the finite sample size and $q_{\max} \approx \pi/\ell$ is an upper cutoff denoting a lower length at which the diffusion approximation breaks down. Although this theory is far from rigorous, it can be shown to agree with the (phenomenological) scaling theory of localization [128]. Moreover, it predicts strong localization, here defined as $D = 0$, to occur in an *infinite* medium when

$$k\ell \approx 1\,; \tag{1.20}$$

this is known as the Ioffe–Regel criterion and was first proposed in the sixties by Mott as a criterion for localization. In a *finite* medium Anderson localization leads to a geometry-dependent diffusion constant and finally to an (ensemble-averaged) transmission that decays exponentially with the size of the medium, not to be confused with the almost trivial exponential decay of the coherent part in (1.6). According to this theory strong-localization phenomena will be demolished in a magnetic field [129]. Unfortunately, this does not agree with exact calculations of the Anderson model with a magnetic field [130] or with random-matrix theory [131]. Furthermore, the observation of light localization in more than one dimension has turned out to be more difficult than suggested by the Ioffe–Regel criterion. At the time of writing, there are reports for microwave localization in two [34] and three [33] dimensions.